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Discrete Light-Cone Quantization in PP-Wave Background

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Abstract

We discuss the discrete light-cone quantization (DLCQ) of a scalar field theory on the maximally supersymmetric pp-wave background in ten dimensions. It has been shown that the DLCQ can be carried out in the same way as in the two-dimensional Minkowski spacetime. Then, the vacuum energy is computed by evaluating the vacuum expectation value of the light-cone Hamiltonian. The results are consistent with the effective potential obtained in our previous work [hep-th/0402028].

Keywords: discrete light-cone quantization, effective potential, pp-wave.

1 Introduction

Recently, the discrete light-cone quantization (DLCQ) method, which has been originally developed by [1, 2](For reviews, see [3, 4]), has a renewed interest related to the M-theory formulation [5] (For a review related to the DLCQ, see [6]). The M-theory formulation [5] has been extended to the pp-wave background [8, 9, 10] by Berenstein-Maldacena-Nastase [11], and so it is also interesting to study the DLCQ method on the pp-wave background. On the other hand, for the type IIB string theory on the pp-wave [12, 13], an interesting work [14] has been done related to the DLCQ in the pp-wave background.

Scalar field theories on pp-wave backgrounds are nice laboratories for studies of the DLCQ method in the pp-wave case. Furthermore these may give interesting cosmological models (For studies of cosmological structure of pp-wave background, see [15, 16, 17, 18, 19]). In fact, scalar field theories on the maximally supersymmetric pp-wave background in ten dimensions are fairly studied. The propagator in this theory was computed in [20], and then the effective potential was discussed in [21] in the path integral formulation [22, 23].

In this letter, we discuss the DLCQ of scalar field theories in the maximally supersymmetric pp-wave background in ten dimensions. The vacuum energies of them are computed by evaluating the vacuum expectation value (VEV) of the light-cone Hamiltonian. The results completely agree with the effective potential calculated in our previous paper [21].

2 DLCQ and Vacuum Energy of a Scalar Field in PP-Wave

We shall consider a scalar field theory on the maximally supersymmetric pp-wave background:

$$ds^2 = -2dx^+dx^- - \sum_{i=1}^{8} \mu^2 (x^i)^2 (dx^+)^2 + \sum_{i=1}^{8} (dx^i)^2 \,, \tag{2.1}$$

$$F_{+1234} = F_{+5678} = 4\mu \,. \tag{2.2}$$

The constant five-form flux F is a field strength of Ramond-Ramond four-form. The light-cone coordinates are defined as $x^{\pm} = (x^0 \pm x^9)/\sqrt{2}\,$.

The action of a scalar field theory on this background is given by

$$\begin{aligned} I_{\rm s} &= \int d^{10}x \sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \\ &= \int d^{10}x \left[\partial_+\phi\partial_-\phi - \frac{1}{2}\mu^2 r^2 \partial_-\phi\partial_-\phi - \frac{1}{2}\partial_i\phi\partial_i\phi - V(\phi)\right] \,, \end{aligned} \tag{2.3}$$

where $r^2 = \sum_{i=1}^{8}(x^i)^2$ and $V(\phi)$ is a potential of a scalar field ϕ. We will concentrate on the case $V(\phi) = m^2\phi^2/2$ below, in order to discuss the DLCQ method for a free scalar field.

The classical equation of motion is

$$\left(\Box_{(\mathrm{pp})} + m^2\right)\phi = 0\,, \qquad \Box_{(\mathrm{pp})} \equiv 2\partial_-\partial_+ - \sum_{i=1}^{8}\partial_i^2 - \mu^2 r^2\partial_-^2\,. \tag{2.4}$$

By using the identity $x\delta(x) = 0$, we can easily find the solution of (2.4):

$$\phi(x) = \sum_{\{n_i\}}\int\frac{dk_-dk_+}{4\pi}\,\delta(2k_+k_- + |k_-|E_n - m^2)\chi_{\{n_i\}}(k_-\,,\ k_+)\,\mathrm{e}^{i(k_+x^+ + k_-x^-)}\prod_{i=1}^{8}\psi_{n_i}(x^i)\,. \tag{2.5}$$

Here we have introduced the following notations:

$$E_n \equiv \sum_{i=1}^{8}E_{n_i} = \mu\sum_{i=1}^{8}\left(n_i + \frac{1}{2}\right) \qquad (i = 1,\ldots,8, \quad n_i = 0,\ldots,\infty)\,,$$

$$\psi_{n_i}(x^i) \equiv N_{n_i}\mathrm{e}^{-\frac{1}{2}\mu|k_-|(x^i)^2}H_{n_i}(\sqrt{\mu|k_-|}x^i)\,, \qquad N_{n_i} \equiv \left(\frac{\mu|k_-|}{\pi}\right)^{1/4}\frac{1}{\sqrt{2^{n_i}n_i!}}\,.$$

The orthonormal and complete conditions:

$$\int_{-\infty}^{\infty}dx\,\psi_m(x)\psi_n(x) = \delta_{mn}\,, \qquad \sum_{n=0}^{\infty}\psi_n(x)\psi_n(y) = \delta(x-y)\,, \tag{2.6}$$

are available to check the normalization of the Poisson bracket.

After some algebra, the classical solution (2.5) can be rewritten as

$$\phi(x) = \frac{1}{4\pi}\sum_{\{n_i\}}\int_0^{\infty}\frac{dk_-}{k_-}\left[a_{\{n_i\}}(k_-)\,\mathrm{e}^{-i(\hat{k}_+x^+ + k_-x^-)} + a^*_{\{n_i\}}(k_-)\,\mathrm{e}^{i(\hat{k}_+x^+ + k_-x^-)}\right]\prod_{i=1}^{8}\psi_{n_i}(x^i)\,, \tag{2.7}$$

where we have defined the on-shell energy as $\hat{k}_+ \equiv (|k_-|\,E_n + m^2)/(2k_-)$ and redefined the coefficients as follows:

$$a_{\{n_i\}}(k_-) \equiv \chi_{\{n_i\}}(k_-,\,\hat{k}_+)\,, \qquad a^*_{\{n_i\}}(k_-) \equiv \chi_{\{n_i\}}(-k_-\,,\ -\hat{k}_+)\,. \tag{2.8}$$

The classical Poisson bracket at the simultaneous light-cone time $x^+ = y^+$ is given by

$$\{\phi(x)\,,\ \pi(y)\}_{\mathrm{P}} = \frac{1}{2}\delta(x^- - y^-)\delta^{(8)}(x^i - y^i)\,, \tag{2.9}$$

where the $\pi(y)$ is the light-cone canonical momentum:

$$\pi \equiv \frac{\partial\mathcal{L}}{\partial(\partial_+\phi)} = \partial_-\phi\,. \tag{2.10}$$

Here the factor 1/2 in the r. h. s. of (2.9) is determined by the Schwinger's action principle and it depends on the convention of the light-cone coordinates* (For the detail, see [4]).

In terms of the coefficients a and a^*, the classical Poisson bracket (2.9) is described as

$$\{a_{\{n_i\}}(k_-)\,,\; a^*_{\{n'_i\}}(k'_-)\}_{\mathrm{P}} = -4\pi i k_- \delta(k_- - k'_-) \prod_{i=1}^{8} \delta_{n_i n'_i}\,. \tag{2.11}$$

It is possible to obtain the usual commutation relation after an appropriate rescaling as we will see later.

In the next we will consider to quantize the theory in the canonical formulation.

2.1 DLCQ Method in PP-Wave Background

Now let us consider the canonical quantization of classical scalar field theories by replacing the classical Poisson bracket of scalar field $\phi(x)$ and the light-cone canonical momentum $\pi(y)$ with the commutator at the simultaneous light-cone time $x^+ = y^+$ (For detail, see [4]). However, we should be careful for the constraint conditions before the replacement of the commutator. In order to carry out the canonical quantization in the light-cone frame, we follow the DLCQ procedure as a standard manner. In this method, the physical system is enclosed in a finite volume with a periodic boundary conditions in x^- and the longitudinal momentum k_- is discretized.

At first, we compactify the light-cone coordinate x^- as $-L \leq x^- \leq L$, and impose periodic boundary condition for the field;

$$\phi(x^+\,,\; x^- = -L\,,\; x^i) = \phi(x^+\,,\; x^- = L\,,\; x^i)\,. \tag{2.12}$$

The corresponding longitudinal momentum is thus discretized as $k_- = \pi q/L$ ($q \in \mathbb{Z}$) and then the Fourier expansion of the field becomes

$$\phi(x) = a_0(x^+) + \sum_{\{n_i\}} \sum_{q>0} \frac{1}{\sqrt{4\pi q}} \left\{ a_{\{n_i\}\,q}(x^+)\, \mathrm{e}^{-i\frac{\pi q}{L}x^-} + a^*_{\{n_i\}\,q}(x^+)\, \mathrm{e}^{i\frac{\pi q}{L}x^-} \right\} \prod_{i=1}^{8} \psi_{n_i,q}(x^i)\,. \tag{2.13}$$

Here after the descritization as

$$\frac{1}{4\pi}\int_0^\infty \frac{dk_-}{k_-} \longrightarrow \frac{1}{4\pi}\sum_{q>0}\frac{1}{q}\,, \qquad k_-\delta(k_- - k'_-) \longrightarrow q\delta_{q,q'}\,,$$

the annihilation and creation operators have been redefined by rescaling as

$$a_{\{n_i\}\,q} \longrightarrow \sqrt{4\pi q}\, a_{\{n_i\}\,q}\,,$$

*If we take the light-cone coordinates as $x^{\pm} = x^0 \pm x^{D-1}$, then the factor 1/2 is removed.

and the plane-wave with x^+ is included in the definition of a and a^*. We also have written the zero mode a_0 separately. Though one may think that the zero-mode naively vanish because of the normalization factors of the Hermite polynomials, the $k_- = 0$ implies a plane-wave expansion rather than a harmonic oscillator one. Hence we still need to treat carefully the zero-mode part. Substituting the expansion (2.13) into the Lagrangian (2.3), we obtain the following expression (discarding a total time derivative)

$$\begin{aligned}
\mathcal{L}\left[a_0\,,\ a_{\{n_i\}\,q}\right] &= \sum_{\{n_i\}}\sum_{q>0}\left\{-ia_{\{n_i\}\,q}\,\dot{a}^*_{\{n_i\}\,q} - \frac{L}{2\pi q}\left(\frac{\pi q}{L}\,E_n + m^2\right)a_{\{n_i\}\,q}\,a^*_{\{n_i\}\,q}\right\} - m^2 L\,a_0^2 \\
&= -i\sum_{\{n_i\}}\sum_{q>0}\ a_{\{n_i\}\,q}\,\dot{a}^*_{\{n_i\}\,q} - H\,,
\end{aligned} \tag{2.14}$$

where the Hamiltonian H is given by

$$H = m^2\,L\,a_0^2 + \sum_{\{n_i\}}\sum_{q>0}\ \frac{L}{2\pi q}\ \left(\frac{\pi q}{L}\,E_n + m^2\right)a_{\{n_i\}\,q}\,a^*_{\{n_i\}\,q}\,, \tag{2.15}$$

and the symbol "$\cdot$" implies the derivative with respect to "x^+", such as $\dot{a}_{\{n_i\}\,q} \equiv da_{\{n_i\}\,q}/dx^+$. From the above, the light-cone Lagrangian is linear with respect to the velocity i.e., a first order system. In this case we can determine the Poisson bracket by comparing the Euler-Lagrange equation and the canonical equation as discussed in [4].

On the one hand, the Euler-Lagrange equations are

$$-\,i\dot{a}_{\{n_i\}\,q} + \frac{L}{2\pi q}\ \left(\frac{\pi q}{L}\,E_n + m^2\right)\,a_{\{n_i\}\,q} \;=\; 0\,, \tag{2.16}$$

$$2m^2 L\,a_0 \;=\; 0\,. \tag{2.17}$$

The second equation (2.17) is non-dynamical and gives a constraint implying the absence of zero mode, i.e. $a_0 = 0$.

On the other hand, using the Hamiltonian (2.15), the canonical equations are

$$\begin{aligned}
\dot{a}_{\{n_i\}\,q} &= \left\{a_{\{n_i\}\,q}\,,\ H\right\}_{\mathrm{P}} \\
&= \sum_{\{n'_i\}}\sum_{l>0}\frac{L}{2\pi l}\ \left(\frac{\pi l}{L}\,E_n + m^2\right)\,a_{\{n'_i\}\,l}\{a_{\{n_i\}\,q}\,,\ a^*_{\{n'_i\}\,l}\}_{\mathrm{P}}\,.
\end{aligned} \tag{2.18}$$

These are identical with the Euler-Lagrange equations (2.16) if we identify the canonical bracket as follows:

$$\{a_{\{n_i\}\,q}\,,\ a^*_{\{n'_i\}\,l}\}_{\mathrm{P}} = -i\delta_{ql}\cdot\prod_{i=1}^{8}\delta_{n_i,n'_i}\,. \tag{2.19}$$

The constraint condition (2.17) can be also derived by differentiating the Hamiltonian as

$$\frac{\partial H}{\partial a_0} = 2m^2 L a_0 = 0\,. \tag{2.20}$$

The quantization of a free scalar field is now performed as usual by replacing the classical Poisson bracket with the commutator as $[\hat{A}\,,\ \hat{B}] = i\,\{A\,,\ B\}_{\rm P}$. As the result, the commutation relation of the creation and annihilation operators is given by

$$[\hat{a}_{\{n_i\}\,q}\,,\ \hat{a}^{\dagger}_{\{n_i'\}\,l}] = \delta_{ql}\cdot\prod_{i=1}^{8}\delta_{n_i,n_i'}\,. \tag{2.21}$$

Finally, the Fock space expansion of the scalar field ϕ becomes

$$\phi\left(x^+\,,\ x^-\,,\ x^i\right) = \sum_{\{n_i\}}\sum_{q>0}\frac{1}{\sqrt{4\pi q}}\left\{\hat{a}_{\{n_i\}\,q}(x^+)\,\mathrm{e}^{-i\frac{\pi q}{L}x^-} + \hat{a}^{\dagger}_{\{n_i\}\,q}(x^+)\,\mathrm{e}^{i\frac{\pi q}{L}x^-}\right\}\prod_{i=1}^{8}\psi_{n_i,q}(x^i)\,. \tag{2.22}$$

As one can see from the expansion (2.22), the Fock space contains only particles with positive longitudinal momentum. Operators with negative longitudinal momentum are annihilation operators. If longitudinal momentum is conserved, positive and discrete, then states with $k_- = q\pi/L$ can have at most q particles among them. Thus, in the sector with q units of momentum, the theory reduces to non-relativistic quantum mechanics including eight harmonic oscillators, with a fixed number of particles.

Finally, it should be remarked that the structure of Fock space in the case of pp-wave background is quite similar to that in two-dimensional Minkowski spacetime. Namely, it is the product of the Fock space of two-dimensional Minkowski and Hermite polynomials. From this result, one can guess that this structure may be extended to other pp-wave cases. In the standard pp-wave cases, it is only the difference that the Hermite polynomials are modified in terms of the oscillation numbers or the number of harmonic oscillators. Remarkably speaking, this structure may be expected to the pp-waves which lead to harmonic oscillators with negative mass terms. We encounter this type of background when Penrose limits of black hole geometries are taken. In these pp-waves, hyperbolic cylinder functions, which imply an instability of the system, would appear instead of Hermite polynomials. However, the hyperbolic cylinder functions do not mean instabilities as already discussed in [16]. On the other hand, our result is also compatible with this fact because the Fock vacuum structure is determined by the light-cone directions only. Namely, it is the two-dimensional Minkowski one.

In the next subsection we will evaluate the vacuum energy of a scalar field by using this operator representation.

2.2 Computation of the Vacuum Energy

Next we shall compute the vacuum energy of a free scalar field in ten-dimensional pp-wave background. In the previous subsection we have discussed the DLCQ of a scalar field and derived the operator expression of the field. We shall compute the vacuum energy density by using this quantized scalar field.

For the Fock vacuum $|0\rangle$ of the light-cone Hamiltonian defined by $a(k_-)|0\rangle = 0$, the vacuum energy density $\mathcal{E}$ is given by[†]

$$\mathcal{E} = \frac{1}{V_9}\langle 0|P_+|0\rangle\,, \qquad V_9 : \text{9-dim. volume}\,, \tag{2.23}$$

where the light-cone Hamiltonian is

$$P_+ = \int_{-\infty}^{\infty} dx^- \int_{-\infty}^{\infty} d^8x \left\{\pi\partial_-\phi - \mathcal{L}[\phi\,,\ \partial_\mu\phi]\right\}. \tag{2.24}$$

If we use the commutation relation (2.21) and the expression (2.22) , the light-cone Hamiltonian is rewritten by

$$P_+ \ = \ \sum_{\{n_i\}}\sum_{q>0} \frac{L}{4\pi q}\left(\frac{\pi q}{L}\,E_n + m^2\right)\left(2\,\hat{a}^\dagger_{\{n_i\}\,q}\,\hat{a}_{\{n_i\}\,q} + 1\right). \tag{2.25}$$

We can evaluate the vacuum energy density $\mathcal{E}$ as

$$\begin{aligned} \mathcal{E} \ &= \ \frac{1}{V_9}\sum_{n=0}^{\infty}{}_{n+7}C_7 \sum_{q>0}\frac{L}{4\pi\, q}\left\{\frac{\pi q}{L}\,|\mu|\,(n+4) + m^2\right\} \\ &= \ \frac{b+1}{8\pi V_8}\,m^2\sum_{q>0}\frac{1}{q}\,, \end{aligned} \tag{2.26}$$

where we have used the $V_9 = 2L\cdot V_8$, the combination factor ${}_{n+7}C_7$ denotes the degeneracy of the sum of n_i , and the coefficient $b+1$ is given by

$$b+1 = \frac{1}{7!}\left\{\zeta_{\rm R}(-7) - 14\zeta_{\rm R}(-5) + 49\zeta_{\rm R}(-3) - 36\zeta_{\rm R}(-1)\right\} = \frac{2497}{3628800}\,. \tag{2.27}$$

Here $\zeta_{\rm R}(s)$ is the Riemann's zeta function. Since the coefficient $b+1$ is computed by the zeta function regularization, there may be the ambiguity for the method of mode sum of n . When the continuum limit is considered by taking $L\to\infty$, we can recover the expression:

$$\mathcal{E} = \frac{b+1}{8\pi V_8}\,m^2\int_0^{\infty} dk_-\,\frac{1}{k_-}\,. \tag{2.28}$$

[†]For computations of vacuum energy of light-cone Hamiltonian in some other models such as Gross-Neveu, $SU(N)$ Thirring, $O(N)$ vector models with large N limit, see [24].

This can be evaluated by introducing the cut-off for the longitudinal momentum as $m^2/\Lambda \leq |k_-| \leq \Lambda$. The resulting vacuum energy density is

$$\mathcal{E} = -\frac{b+1}{8\pi V_8}\, m^2 \,\ln\left(\frac{m^2}{\Lambda^2}\right) , \tag{2.29}$$

where the factor $b+1$ denotes the quantum correction which comes from the regularization of mode sum of n. Note that the contribution of the factor $b+1$ does not appear in the tree level calculation [25, 26]. This result agrees with the 1-loop effective potential with $V(\phi) = m^2\phi^2/2$ that is obtained by using the path integral method [21].

On the other hand, we may consider the limit $L \to \infty$ before the concrete computation, and introduce the cut-off for the longitudinal momentum as $m^2/\Lambda \leq |k_-| \leq \Lambda$. The $k_- = 0$ seems to have a subtlety, but this mode decouple from the theory as already discussed and so this point may have no problem. After taking the decompactification limit, we use the the expansion of scalar field (2.7) with the creation and annihilation operators whose commutation relations are

$$[\hat{a}_{\{n_i\}}(k_-)\,,\ \hat{a}^\dagger_{\{n'_i\}}(k'_-)] = 4\pi k_- \delta(k_- - k'_-) \prod_{i=1}^{8} \delta_{n_i n'_i} \,. \tag{2.30}$$

Thus, the light-cone Hamiltonian P_+ is given by using the expressions (2.7) and (2.30)

$$P_+ \;=\; \frac{1}{2}\delta(0) \sum_{\{n_i\}} \int_{m^2/\Lambda}^{\Lambda} \frac{dk_-}{k_-} \left\{ \mu\, k_- \sum_{i=1}^{8} \left(n_i + \frac{1}{2} \right) + m^2 \right\} . \tag{2.31}$$

By using the relation $\delta(0)/V_9 = (2\pi)V_1/V_9 = (2\pi)/V_8$, we can rederive the vacuum energy density (2.29) again.

Notably, the effective potential is independent of the parameter μ. We can see this fact by noting that the effect of μ (i.e., E_n) can be absorbed by shifting the light-cone momentum k_+. But physically, as we discussed in [21], the vacuum energies produced by the quantum fluctuations flow from transverse space to the k_+-direction due to the flux equipped with the pp-wave geometry. As the result of the energy flow, the effect of the pp-wave background would be realized only as the numerical coefficients, and thus the vacuum energy may not explicitly depend on the parameter μ.

3 A Conclusion and Discussions

We have discussed the DLCQ of a scalar field theory in the maximally supersymmetric pp-wave background in ten dimensions. The DLCQ method in the pp-wave background has been shown

to work well in the same way as in two-dimensional case, even if we consider higher dimensional theory. It is because transverse momenta are discretized and so the treatment in the pp-wave case is quite similar to the two-dimensional Minkowski case.

We also calculated the vacuum energy of a free scalar field in the pp-wave and it has been shown the resulting vacuum energy surely agrees with the effective potential obtained in our previous paper.

The DLCQ method may play an important role in the M-theory formulation [5], where the discrete light-cone quantized M-theory can be described by a matrix model. It would be an important key ingredient in studies of the pp-wave matrix model [11]. We believe that our study of DLCQ in a scalar field theory on the pp-wave background should be a clue to shed light on some features of DLCQ method in the pp-wave case.

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